

Zero-Mode Contribution in Nucleon-Delta Transition

Jianghao Yu and Teng Wang

Department of Physics, Peking University, Beijing 100871, China

Chueng-Ryong Ji

Department of Physics, North Carolina State University, Raleigh, NC 27695-8202, USA

Bo-Qiang Ma

Department of Physics, Peking University, Beijing 100871, China and

MOE Key Laboratory of Heavy Ion Physics, Peking University, Beijing 100871, China

We investigate the transition form factors between nucleon and $\Delta(1232)$ particles by using a covariant quark-spectator-diquark field theory model in (3+1) dimensions. Performing a light-front calculation in parallel with the manifestly covariant calculation in light-front helicity basis, we examine the light-front zero-mode contribution to the helicity components of light-front good (“+”) current matrix elements. Choosing the light-front gauge ($\epsilon_{h=\pm}^+ = 0$) with circular polarization in Drell-Yan-West frame, we find that only the helicity components $(\frac{1}{2}, \frac{1}{2})$ and $(\frac{1}{2}, -\frac{1}{2})$ of the good current receive the zero-mode contribution. Taking into account the zero-mode, we find the prescription independence in obtaining the light-front solution of form factors from any three helicity matrix elements with smeared light-front wavefunctions. The angular condition, which guarantees the full covariance of different schemes, is recovered.

I. INTRODUCTION

The study of nucleon transition form factors has long been recognized as one of the essential steps towards a deep understanding of strong interaction. The transition process between nucleon and $\Delta(1232)$ resonance, the lowest baryon excitation, is important to investigate the internal quark and gluon structure of the nucleon and its resonance. When hadronic systems are described in terms of quarks and gluons, it is part of nature that the characteristic momenta are of the same order or even very much larger than that of the masses of the particles involved. Thus, a relativistic treatment is one of the essential ingredients for a successful model description. The light-front constituent quark model (LFQM) within the framework of light-front (LF) dynamics [1] appears to be a useful phenomenological tool which keeps the relativistic effect into account while it incorporates some basic spin-flavor structure of the hadron.

In the formalism of LFQM, the good (“+”) component of current matrix elements for the analysis of the hadron form factors can be divided into two parts. One is the valence part, in which the “+” current matrix elements are diagonal in the Fock state expansion, that can be expressed in terms of the convolutions of LF wavefunctions. The other is the nonvalence part, in which the “+” com-

ponents are only related to the off-diagonal elements in the Fock state expansion. In the $q^+ \rightarrow 0$ limit, where q^+ is the longitudinal component of the momentum transfer q ($q^\pm = q^0 \pm q^3$, $q^2 = q^+q^- - \vec{q}_\perp^2$), it was usually taken for granted that the nonvalence part vanishes and thus one only needs to take into account the valence part or the diagonal elements in the Fock state expansion. This is true in the cases when the hadrons involved are spin-0 bosons and spin-1/2 fermions as in most previous investigations. The “+” component of matrix elements is therefore denoted as “good” component. However, in recent analyses of spin-1 form factors [2, 3, 4, 5], it has been shown that the nonvalence part has non-vanishing contribution in the $q^+ \rightarrow 0$ limit, which is called *zero mode*. Without the zero mode contribution, the angular condition [6], which assures that different ways of extracting the form factors produce the same physical results, would be violated.

In this paper we analyze the zero mode contribution to the transition form factors in the N - Δ transition process. As our purpose is for a recovery of full covariance in the LFQM formalism by including the zero-mode contribution in the N - Δ process but not to present a realistic model which can fit the available data, we adopt a simple but exactly solvable quark-spectator-diquark model for the sake of simplicity and clarification. We specify the

kinematics in Drell-Yan-West (DYW) frame with the LF gauge ($\epsilon_{h=\pm}^+ = 0$), which has been used in the LFQM analysis. To investigate the zero mode contribution in a clear way, we make the taxonomical decomposition of the full current matrix elements into valence and nonvalence contributions. Performing calculation in the LF helicity basis we find that indeed the good component of the currents has non-vanishing zero mode contribution in this model. In particular, there is zero mode contribution in the LF helicity amplitudes $(h', h) = (\frac{1}{2}, \frac{1}{2})$ and $(\frac{1}{2}, -\frac{1}{2})$, where h and h' are the initial and final helicities, respectively. When the number of current matrix elements gets larger than the number of physical form factors, the conditions that the current matrix elements must satisfy are essential to test the underlying theoretical model for the hadrons. The analysis of N - Δ transition process requires in general eight light-front helicity amplitudes although there are only three physical form factors. Thus, there must be five conditions for the helicity amplitudes. Using the light-front parity relation, one can reduce the number of helicity amplitudes down to four. One more relation among the remaining four helicity amplitudes comes from the conservation of angular momentum and this relation is called *angular condition*. Consequently, only three helicity amplitudes are independent of each other as it should be because there are only three physical form factors involved in the N - Δ transition process. However, there are several different prescriptions [7, 8] in choosing independent current matrix elements to extract the three physical form factors. We take four prescriptions as examples, and show that different types of helicity combinations produce the same results and the angular condition is satisfied if the zero mode contribution is included in each description. We do this taking the baryon as a system of quark and diquark. Note that there are different ways of treating the diquarks in the literature. One way is called the quark-spectator-diquark model in which one would re-organize the spectators by an effective diquark to take into account the flavor-spin structure of the whole spectator when any of the quark is struck. In this framework, there is no struck diquark and one should have an additional factor of 3 to account for that there are three valence quarks in the baryon. Another way is called the quark-diquark model in which the diquarks are treated as independent particles and one should also take into account the contribution when diquarks are struck. However, the zero-mode issue is completely independent from whether the model is “spectator diquark” or “struck

diquark”. That is to say that the Lorentz covariance must work independently for each contribution. In this work, we thus compute only one part of the full contribution neglecting the flavor structure and show that the covariance is nevertheless recovered on its own with the inclusion of the zero-mode contribution. The struck diquark contribution would independently work essentially in the same way.

This paper is organized as follows. In Section II, we present the Lorentz-invariant transition form factors and angular conditions in LF helicity basis. The four prescriptions used in extracting the physical form factors are also briefly discussed. In Section III, after taking into account the Melosh transformation [9] for instant-form SU(6) quark-diquark wave functions, we get effective covariant vertices for the nucleon and Δ coupling with the quark and the diquark. In Section IV, we present our model calculations in both the manifestly covariant Feynman method and the LF technique. In the $q^+ = 0$ frame, we separate the full amplitudes to the valence and non-valence parts to determine zero mode contribution by a power counting method. In Section V, we present the numerical results of transition form factors and angular condition. Conclusions are given in Section VI. In Appendix, we summarize our results of the LF helicity amplitudes for each helicity component used in our calculation.

II. TRANSITION FORM FACTOR IN LIGHT-FRONT HELICITY BASIS

The N - Δ transition matrix element of the electromagnetic current J^μ between the initial $|p, h\rangle$ and the final $|p', h'\rangle$ states can be written as

$$J_{h'h}^\mu = \langle p', h' | J^\mu | p, h \rangle = \bar{u}_\rho(p', h') \Gamma^{\rho\mu} u(p, h), \quad (1)$$

where $u(p, h)$ is the nucleon Dirac spinor with momentum p and helicity h , and $u_\nu(p', h')$ is the Rarita-Schwinger spin- $\frac{3}{2}$ spinor for the Δ particle with momentum p' and helicity h' , respectively. Here the covariant tensor $\Gamma^{\rho\mu}$ is defined by a linear combination of three kinematic-singularity-free transition form factors $G_1(Q^2)$, $G_2(Q^2)$ and $G_3(Q^2)$:

$$\begin{aligned} \Gamma^{\mu\nu} = & G_1(Q^2) \left(q^\mu \gamma^\nu - \gamma \cdot q g^{\mu\nu} \right) \gamma_5 \\ & + G_2(Q^2) \left(q^\mu \frac{(p+p')^\nu}{2} - \frac{(p+p') \cdot q}{2} g^{\mu\nu} \right) \gamma_5 \\ & + G_3(Q^2) \left(q^\mu q^\nu - q^2 g^{\mu\nu} \right) \gamma_5, \end{aligned} \quad (2)$$

where $q^\mu = (p' - p)^\mu$ is the four-momentum transfer and $Q^2 = -q^2$.

The physical Sachs-like form factors with definite multipole or helicity, such as magnetic dipole $G_M(Q^2)$,

electric quadrupole $G_E(Q^2)$ and Coulomb quadrupole $G_C(Q^2)$ form factors, are related in a well-known way to the three kinematic-singularity-free transition form factors [10, 11] as follows

$$\begin{aligned} G_M(Q^2) &= \frac{M_P}{3(M_P + M_\Delta)} \left[\frac{(3M_\Delta + M_P)(M_\Delta + M_P) + Q^2}{M_\Delta} G_1(Q^2) + (M_\Delta^2 - M_P^2) G_2(Q^2) - 2Q^2 G_3(Q^2) \right], \\ G_E(Q^2) &= \frac{M_P}{3(M_P + M_\Delta)} \left[\frac{M_\Delta^2 - M_P^2 - Q^2}{M_\Delta} G_1(Q^2) + (M_\Delta^2 - M_P^2) G_2(Q^2) - 2Q^2 G_3(Q^2) \right], \\ G_C(Q^2) &= \frac{M_P}{3(M_\Delta + M_P)} \left[4M_\Delta G_1(Q^2) + (3M_\Delta^2 + M_P^2 + Q^2) G_2(Q^2) + 2(M_\Delta^2 - M_P^2 - Q^2) G_3(Q^2) \right], \end{aligned} \quad (3)$$

where M_Δ and M_P are the masses of $\Delta(1232)$ and nucleon.

The reference frame is taken as the Drell-Yan-West (DYW) frame [12] where $q^+ = 0$ and $q^2 = q^+q^- - \vec{q}_\perp^2 < 0$. In this frame, the momenta of the nucleon and $\Delta(1232)$ particle are assigned as

$$\begin{aligned} q^\mu &= \left(0, \frac{\vec{q}_\perp^2 + M_\Delta^2 - M_P^2}{p^+}, \vec{q}_\perp \right), \\ p^\mu &= \left(p^+, \frac{M_P^2}{p^+}, \vec{0}_\perp \right), \\ p'^\mu &= \left(p^+, \frac{\vec{q}_\perp^2 + M_\Delta^2}{p^+}, \vec{q}_\perp \right). \end{aligned} \quad (4)$$

Here, we use the notation $p = (p^+, p^-, p^1, p^2)$ and the metric convention $p^2 = p^+p^- - \vec{p}_\perp^2$ with $p^\pm = p^0 \pm p^3$.

For the spin- $\frac{3}{2}$ particle such as $\Delta(1232)$, in the LF formalism the Rarita-Schwinger spinor is

$$\begin{aligned} u^\mu(p^+, \vec{p}_\perp, \frac{3}{2}) &= \epsilon^\mu(p^+, \vec{p}_\perp, +1) u(p^+, \vec{p}_\perp, \uparrow), \\ u^\mu(p^+, \vec{p}_\perp, \frac{1}{2}) &= \sqrt{\frac{2}{3}} \epsilon^\mu(p^+, \vec{p}_\perp, 0) u(p^+, \vec{p}_\perp, \uparrow) \\ &\quad + \sqrt{\frac{1}{3}} \epsilon^\mu(p^+, \vec{p}_\perp, +1) u(p^+, \vec{p}_\perp, \downarrow). \end{aligned} \quad (5)$$

Following Bjorken-Drell convention, $u(p^+, \vec{p}_\perp, h)$ is the light-front spinor with light-front momentum (p^+, \vec{p}_\perp) and helicity h and $\epsilon^\mu(p^+, \vec{p}_\perp, h)$ is the transverse ($h = \pm$) or longitudinal ($h = 0$) polarization vector with the convention $\epsilon^\mu = (\epsilon^+, \epsilon^-, \epsilon^1, \epsilon^2)$. With $\epsilon^+(p, \pm) = 0$ from the LF gauge, the general form of the LF polarization vector

is given by

$$\begin{aligned} \epsilon^\mu(p, \vec{p}_\perp, \pm) &= \left(0, \frac{2\vec{\epsilon}_\perp(\pm) \cdot \vec{p}_\perp}{p^+}, \vec{\epsilon}_\perp(\pm) \right), \\ \epsilon^\mu(p^+, \vec{p}_\perp, 0) &= \frac{1}{M_\Delta} \left(p^+, \frac{\vec{p}_\perp^2 - M_\Delta^2}{p^+}, p^1, p^2 \right), \end{aligned} \quad (6)$$

which satisfies $p \cdot \epsilon(\pm) = 0$ with $\vec{\epsilon}_\perp(\pm) = \mp \frac{1}{\sqrt{2}}(1, \pm i)$.

In DYW frame, the covariant form factors can be determined using only the plus component of the current, $J_{h'h}^+(0) \equiv \langle P', h' | J^+ | P, h \rangle$. As one can see from Eq. (2), eight matrix elements of $J_{h'h}^+$ can be assigned to the current operator, while there are only three independent invariant form factors. However, according to LF discrete symmetry, the current matrix elements $J_{h'h}^+$ must be constrained by the invariance under the LF parity [13]

$$J_{-h', -h}^+ = \eta'_\Delta \eta_P (-1)^{S' - S + h' - h} J_{h', h}^+, \quad (7)$$

where η'_Δ and η_P are the intrinsic parity of $\Delta(1232)$ with spin S' and nucleon with spin S . Since the initial and final states are different in N - Δ transition, the LF time-reversal condition does not reduce the number of matrix elements but requires $J_{h'h}^+$ to be real. Therefore, one can reduce the eight matrix elements of the plus current $J_{h'h}^+$ down to four using the LF helicity basis, and we take $J_{\frac{3}{2}, \frac{1}{2}}^+$, $J_{\frac{3}{2}, -\frac{1}{2}}^+$, $J_{\frac{1}{2}, \frac{1}{2}}^+$ and $J_{\frac{1}{2}, -\frac{1}{2}}^+$. The plus components of current matrix elements are related to the kinematic-singularity-free form factors $G_{1,2,3}$ with the following four

relations

$$\begin{aligned}
J_{\frac{3}{2}, \frac{1}{2}}^+ &= \sqrt{2} q^L [2G_1 + (M_\Delta - M_P)G_2], \\
J_{\frac{1}{2}, \frac{1}{2}}^+ &= \sqrt{\frac{2}{3}} \frac{Q^2}{M_\Delta} [-2G_1 - (2M_\Delta - M_P)G_2 \\
&\quad + 2(M_\Delta - M_P)G_3], \\
J_{\frac{1}{2}, -\frac{1}{2}}^+ &= \sqrt{\frac{2}{3}} \frac{q^L}{M_\Delta} [2M_P G_1 - 2Q^2 G_3 \\
&\quad + (Q^2 - (M_\Delta - M_P)M_\Delta)G_2], \\
J_{\frac{3}{2}, -\frac{1}{2}}^+ &= -\sqrt{2}(q^L)^2 G_2.
\end{aligned} \tag{8}$$

From above relations, it is rather obvious that the four helicity components are not independent because there are only three physical form factors to be extracted. Thus, there must be an additional constraint on the current matrix elements. Indeed, the rotation invariance of the system (angular momentum conservation) offers an additional constraint on the current operator, which yields the so called “angular condition” $\Delta(Q^2)$ [13] given by

$$\begin{aligned}
\Delta(Q^2) &= q^L [Q^2 - M_P(M_\Delta - M_P)] J_{\frac{3}{2}, \frac{1}{2}}^+ \\
&\quad + \sqrt{3}(q^L)^2 M_\Delta J_{\frac{1}{2}, \frac{1}{2}}^+ \\
&\quad + \sqrt{3} q^L M_\Delta (M_\Delta - M_P) J_{\frac{1}{2}, -\frac{1}{2}}^+ \\
&\quad - [(M_\Delta - M_P)(M_\Delta^2 - M_P^2) + M_P Q^2] J_{\frac{3}{2}, -\frac{1}{2}}^+ \\
&= 0.
\end{aligned} \tag{9}$$

Because of the angular condition, only three helicity amplitudes are independent as expected. However, the relations between the physical form factors and the matrix elements $J_{h'h}^+$ are not uniquely determined because the number of helicity amplitudes is larger than that of physical form factors. So one still has the freedom of choice in extracting the form factors by using different matrix elements. For example, Weber [7] and Cardarelli *et al.* [8] used helicity components $(h', h) = (\frac{3}{2}, \frac{1}{2}), (\frac{1}{2}, \frac{1}{2})$ and $(\frac{1}{2}, -\frac{1}{2})$, which is denoted as “Scheme A” for this prescription. In this scheme, the kinematic-singularity-free transition form factors in terms of the matrix elements

$J_{h'h}^+$ are given by

$$\begin{aligned}
G_1^A &= \frac{M_\Delta}{q^L K_A} \left[((M_\Delta - M_P)^2 + Q^2) J_{\frac{3}{2}, \frac{1}{2}}^+ \right. \\
&\quad \left. + \sqrt{3}(M_\Delta - M_P)^2 J_{\frac{1}{2}, -\frac{1}{2}}^+ \right. \\
&\quad \left. + \sqrt{3} q^L (M_\Delta - M_P) J_{\frac{1}{2}, \frac{1}{2}}^+ \right], \\
G_2^A &= \frac{2}{q^L K_A} \left[((M_\Delta - M_P)M_P - Q^2) J_{\frac{3}{2}, \frac{1}{2}}^+ \right. \\
&\quad \left. - \sqrt{3}(M_\Delta - M_P)M_\Delta J_{\frac{1}{2}, -\frac{1}{2}}^+ \right. \\
&\quad \left. - \sqrt{3}M_\Delta q^L J_{\frac{1}{2}, \frac{1}{2}}^+ \right], \\
G_3^A &= \frac{1}{q^L K_A} \left[(M_\Delta^2 + M_\Delta M_P - M_P^2 - Q^2) J_{\frac{3}{2}, \frac{1}{2}}^+ \right. \\
&\quad \left. + \sqrt{3} q^L M_\Delta (M_\Delta^2 - M_P^2 - Q^2) J_{\frac{1}{2}, \frac{1}{2}}^+ / Q^2 \right. \\
&\quad \left. - \sqrt{3}M_\Delta^2 J_{\frac{1}{2}, -\frac{1}{2}}^+ \right],
\end{aligned} \tag{10}$$

where $K_A = 2\sqrt{2}(M_\Delta - M_P)(M_\Delta^2 - M_P^2) + 2\sqrt{2}M_P Q^2$. Keeping the $(\frac{1}{2}, \frac{1}{2})$ component that gives the most dominant contribution in the high momentum region, we can use helicity $(\frac{3}{2}, \frac{1}{2}), (\frac{1}{2}, \frac{1}{2})$, and $(\frac{3}{2}, -\frac{1}{2})$ amplitudes to determine covariant form factors, which we call “Scheme B”, as follows

$$\begin{aligned}
G_1^B &= \frac{1}{2\sqrt{2}q^L} J_{\frac{3}{2}, \frac{1}{2}}^+ + \frac{M_\Delta - M_P}{2\sqrt{2}(q^L)^2} J_{\frac{3}{2}, -\frac{1}{2}}^+, \\
G_2^B &= -\frac{1}{\sqrt{2}(q^L)^2} J_{\frac{3}{2}, -\frac{1}{2}}^+, \\
G_3^B &= \frac{1}{2\sqrt{2}(M_\Delta - M_P)} \left[\frac{1}{q^L} J_{\frac{3}{2}, \frac{1}{2}}^+ + \frac{\sqrt{3}M_\Delta}{Q^2} J_{\frac{1}{2}, \frac{1}{2}}^+ \right. \\
&\quad \left. - \frac{M_\Delta}{(q^L)^2} J_{\frac{3}{2}, -\frac{1}{2}}^+ \right].
\end{aligned} \tag{11}$$

Alternatively we can avoid using the helicity $(\frac{1}{2}, \frac{1}{2})$ but use helicity $(\frac{3}{2}, \frac{1}{2}), (\frac{1}{2}, -\frac{1}{2})$, and $(\frac{3}{2}, -\frac{1}{2})$ amplitudes (“Scheme C”)

$$\begin{aligned}
G_1^C &= \frac{1}{2\sqrt{2}q^L} J_{\frac{3}{2}, \frac{1}{2}}^+ + \frac{M_\Delta - M_P}{2\sqrt{2}(q^L)^2} J_{\frac{3}{2}, -\frac{1}{2}}^+, \\
G_2^C &= -\frac{1}{\sqrt{2}(q^L)^2} J_{\frac{3}{2}, -\frac{1}{2}}^+, \\
G_3^C &= \frac{1}{2\sqrt{2}Q^2} \left[\frac{M_P}{q^L} J_{\frac{3}{2}, \frac{1}{2}}^+ - \frac{\sqrt{3}M_\Delta}{q^L} J_{\frac{1}{2}, -\frac{1}{2}}^+ \right. \\
&\quad \left. + \frac{M_\Delta^2 - M_P^2 - Q^2}{(q^L)^2} J_{\frac{3}{2}, -\frac{1}{2}}^+ \right].
\end{aligned} \tag{12}$$

From above schemes, we find that the only difference between “Scheme B” and “Scheme C” is the expression of $G_3(Q^2)$. If we take $G_3^B = G_3^C$, we can recover the angular condition given by Eq. (9). In the above schemes A, B and C, the helicity $(\frac{3}{2}, -\frac{1}{2}), (\frac{1}{2}, -\frac{1}{2})$ and $(\frac{1}{2}, \frac{1}{2})$ are

avoided in Eqs. (10), (11) and (12), respectively. Since there is one more helicity ($\frac{3}{2}, \frac{1}{2}$) that can be avoided, it is natural to consider another scheme using helicity components ($\frac{3}{2}, -\frac{1}{2}$), ($\frac{1}{2}, \frac{1}{2}$), and ($\frac{1}{2}, -\frac{1}{2}$) and get “Scheme D”:

$$\begin{aligned} G_1^D &= \frac{\sqrt{3}M_\Delta}{K_D} \left[J_{\frac{1}{2}, \frac{1}{2}}^+ + \frac{(M_\Delta - M_N)}{q^L} J_{\frac{1}{2}, -\frac{1}{2}}^+ \right. \\ &\quad \left. - \frac{(M_\Delta - M_N)^2 + Q^2}{\sqrt{3}(q^L)^2} J_{\frac{3}{2}, -\frac{1}{2}}^+ \right], \\ G_2^D &= -\frac{\sqrt{2}}{2(q^L)^2} J_{\frac{3}{2}, -\frac{1}{2}}^+, \\ G_3^D &= \frac{\sqrt{3}}{K_D} \left[\frac{M_\Delta M_N}{Q^2} J_{\frac{1}{2}, \frac{1}{2}}^+ + \frac{M_\Delta}{q^L} J_{\frac{1}{2}, -\frac{1}{2}}^+ \right. \\ &\quad \left. + \frac{M_\Delta^2 + Q^2 - M_N^2 - M_\Delta M_N}{\sqrt{3}(q^L)^2} J_{\frac{3}{2}, -\frac{1}{2}}^+ \right], \quad (13) \end{aligned}$$

where $K_D = 2[(M_\Delta - M_N)M_N - Q^2]$. If we take $G_2^A = G_2^D$ in Eqs. (11) and (13), then we also get the “angular condition” $\Delta(Q^2)$. Of course, we also have other choices that involve all four helicity amplitudes in various ways. We will show later that different schemes are equivalent and give the same physical result by taking into account the zero mode contribution.

III. QUARK-SPECTATOR-DIQUARK MODEL DESCRIPTION

In light-front dynamics, the set of LF wavefunctions provides a frame-independent description of hadrons in Fock state expansion. In the light-front quark model, the LF wavefunction of a composite system can be obtained by transforming ordinary equal-time (instant-form) wavefunction in the rest frame into LF wavefunction. This can be done by taking into account relativistic effects such as the Melosh rotation [9]. For the lowest excited baryon such as proton and $\Delta(1232)$, a convenient Fock state basis is the quark-diquark two-body state, which has been adopted in Refs. [14, 15, 16].

The proton spin wavefunction in the SU(6) quark-diquark model in the instant-form can be written as [17]

$$\begin{aligned} \Psi_P^{\pm\frac{1}{2}}(qD) &= \pm\frac{1}{3} \left[V^0(ud)u^{\uparrow, \downarrow} - \sqrt{2}V^{\pm 1}(ud)u^{\downarrow, \uparrow} \right. \\ &\quad \left. - \sqrt{2}V^0(uu)d^{\uparrow, \downarrow} + 2V^{\pm 1}(uu)d^{\downarrow, \uparrow} \right] \\ &\quad + S(ud)u^{\uparrow, \downarrow}, \quad (14) \end{aligned}$$

where \uparrow, \downarrow label the spin projection $J_q^z = \pm\frac{1}{2}$ of the quark, $V^{sz}(q_1q_2)$ stands for the q_1q_2 axial vector diquark with the third spin component s_z , and $S(ud)$ stands for a ud

scalar diquark. In the inclusive and exclusive processes involving above quark-diquark wavefunction, the diquark serves as an effective spectator to account for the spin-flavor structure of whole spectators and do not serve as a struck particle to contribute to the electro-magnetic properties [14, 16, 17]. This is to say, when any of the quarks is struck, we just take it as the quark and the spectators are treated as an effective diquark, and we call this kind of description as the quark-spectator-diquark model. Alternatively, when one takes another kind of proton SU(6) spin wavefunction such as in Ref. [18], the diquark is treated as an independent particle which can be a struck particle. In such a model, one would not get the right electro-magnetic properties in exclusive processes unless the struck diquark contribution is included. In this work we aim at the generic structure of zero mode contribution and take into account only the quark-spectator-diquark contribution neglecting the flavor structure in the above two kinds of instant-form wave functions.

Without the flavor structure, the proton SU(6) spin wavefunctions in Eq. (14) and Ref. [18] can be expressed as [19, 20]

$$\begin{aligned} \Psi_P^S(qD) &= \sum_{s_1, s_2, s_3} (\chi_{s_1}^\dagger \vec{\sigma}_1 \sigma_2 \chi_{s_2}^*) \cdot (\chi_{s_3}^\dagger \vec{\sigma} \chi_s) V^{s_1+s_2}(q_1q_2) q_3^{s_3} \\ &\quad + \sum_{s_1, s_2, s_3} (\chi_{s_1}^\dagger i\sigma_2 \chi_{s_2}^*) (\chi_{s_3}^\dagger \chi_s) S^0(q_1q_2) q_3^{s_3}, \quad (15) \end{aligned}$$

where χ_s is the instant-form Dirac spinor with helicity s in the rest frame, and $V^{s_1+s_2}(q_1q_2) q_3^{s_3}$ and $S^0(q_1q_2) q_3^{s_3}$ are given in the instant-form spin basis. We transform the instant spinor χ_s into the LF Dirac spinor $u_{LF}(\lambda)$ with LF helicity λ by the Melosh transformation [9]

$$\chi_s = u_{LF}(\lambda) R_{s, \lambda}, \quad (16)$$

where

$$R_{s, \lambda} = \frac{1}{\sqrt{2k^+(k^0 + m)}} \begin{pmatrix} k^+ + m & k^1 - ik^2 \\ -k^1 - ik^2 & k^+ + m \end{pmatrix}. \quad (17)$$

After the Melosh transformation, the proton spin wavefunction in LF helicity basis is given by

$$\begin{aligned} \Psi_P^h(qD)_{LF} &= (\bar{u}_1 \gamma^\mu \vec{\tau} G \bar{u}_2^T) \cdot (\bar{u}_3 \gamma_\mu \gamma_5 \vec{\tau} u_P) V^{\lambda_1+\lambda_2}(q_1q_2) q_3^{\lambda_3} \\ &\quad + (\bar{u}_1 \gamma_5 G \bar{u}_2^T) (\bar{u}_3 u_P) S^0(q_1q_2) q_3^{\lambda_3}, \quad (18) \end{aligned}$$

where $G = i\tau_2 C = G^T$ is the G parity with the charge-conjugation operator $C = i\gamma^2 \gamma^0$ and $u_i = u_i(\lambda)$ are the LF spinors. We now recognize $\bar{u}_1 \gamma^\mu \vec{\tau} G \bar{u}_2^T$ as the axial vector diquark ε_ν^μ and $\bar{u}_1 \gamma_5 G \bar{u}_2^T$ as the scalar diquark ϕ_s .

Thus, we can rewrite the LF proton spin wavefunctions in Eq. (18), modulo isospin terms, as

$$|P, h\rangle = \bar{u}(\lambda_q)\gamma \cdot \varepsilon_V(\lambda_V)\gamma_5 u_P(h) |V, q; \lambda_V, \lambda_q\rangle + \phi_S \bar{u}(\lambda_q) u_P(h) |S, q; \lambda_S, \lambda_q\rangle, \quad (19)$$

where $|P, h\rangle = \Psi_P^{\lambda_0}(qD)_{LF}$ and $|D, q; \lambda_D, \lambda_q\rangle = D^{\lambda_D}(q_1 q_2) q_3^{\lambda_q}$ are the Fock states with D denoting the vector or scalar diquark. Consequently, we can introduce the effective quark-diquark-nucleon vertex, $\bar{u}(\lambda_q)\gamma \cdot \varepsilon_V(\lambda_V)\gamma_5 u_P(\lambda_0)$ for the axial vector diquark and $\phi_S \bar{u}(\lambda_q) u_P(\lambda_0)$ for the scalar diquark.

The $\Delta(1232)$ wavefunctions in the $SU(6)$ quark-diquark model are

$$\begin{aligned} \Psi_{\Delta^+}^{\pm \frac{1}{2}}(qD) &= \pm \frac{1}{3} \left[2V^0(ud) u^{\uparrow, \downarrow} + \sqrt{2}V^{\pm 1}(ud) u^{\downarrow, \uparrow} \right. \\ &\quad \left. + \sqrt{2}V^0(uu) d^{\uparrow, \downarrow} + V^{\pm 1}(uu) d^{\downarrow, \uparrow} \right], \\ \Psi_{\Delta^+}^{\pm \frac{3}{2}}(qD) &= \pm \frac{1}{\sqrt{3}} \left[\sqrt{2}V^{\pm 1}(ud) u^{\uparrow, \downarrow} + V^{\pm 1}(uu) d^{\uparrow, \downarrow} \right]. \end{aligned} \quad (20)$$

After similar treatment, we can take the effective quark-diquark-delta vertex as $\bar{u}(\lambda_q)\varepsilon_V^\mu(\lambda_V)u_\mu(\lambda_0)$ for the axial vector diquark. Here $u_\mu(\lambda_0)$ is the light-front Rarita-Schwinger spinor defined in Eq. (5). In the Δ case, there is no coupling with the scalar diquark as shown in Eq.(20).

In the above quark-diquark two-body Fock state basis, the proton and $\Delta(1232)$ spin LF wavefunctions can be expressed by the full relativistic effective vertices. This covariant description provides us a convenient tool to treat the diagonal and off-diagonal elements of the plus current in the lowest Fock state expansion.

IV. LIGHT-FRONT CALCULATION IN A SOLVABLE COVARIANT MODEL

From the above fully relativistic description of the nucleon and $\Delta(1232)$ spin wavefunctions, we can take an effective treatment of the N - Δ electromagnetic transition from the covariant field theory. Similar to the description in the vector meson case [2, 3, 4, 5], a solvable model based on the covariant model of $(3+1)$ -dimensional field theory enables us to derive the form factors in LF helicity basis.

The matrix element $J_{h'h}^\mu(0)$ of the electromagnetic current with constituents of masses m_q and m_d obtained

from the covariant diagram of Fig. 1(a) is given by

$$\begin{aligned} J_{h'h}^\mu(0) &= iN_c g_\Delta g_P \int \frac{d^4 k}{(2\pi)^4} \frac{S_\Lambda(k-p) S_{h'h}^\mu S_\Lambda(k-p')}{[k^2 - m_d^2 + i\varepsilon]} \\ &\quad \times \frac{1}{[(k-p)^2 - m_q^2 + i\varepsilon][(k-p')^2 - m_q^2 + i\varepsilon]}, \end{aligned} \quad (21)$$

where g_Δ and g_P are the coupling constants and N_c is the number of colors. In Eq.(21), $S_{h'h}^\mu$ is the spinor matrix element

$$\begin{aligned} S_{h'h}^\mu &= \bar{u}_\rho(p', h')(\not{p}' - \not{k} + m)\gamma^\mu(\not{p} - \not{k} + m)\gamma_\nu \gamma_5 u(p, h) \\ &\quad \times \sum_{\lambda_V} \epsilon^\rho(k, \lambda_V) \epsilon^{*\nu}(k, \lambda_V), \end{aligned} \quad (22)$$

where $\epsilon^\rho(k, \lambda_V)$ is the polarized vector for axial vector diquark with helicity λ_V . To regularize the covariant triangle-loop in $(3+1)$ dimension, we replace the point-like photon-vertex γ^μ by a non-local (smeared) photon-vertex $S_\Lambda(p'-k)\gamma^\mu S_\Lambda(p-k)$, where $S_\Lambda(p-k) = \Lambda^2/((k-p)^2 - \Lambda^2 + i\varepsilon)$ and Λ plays the role of a momentum cut-off similar to the Pauli-Villars regularization [21].

From Eqs. (8) and (21), we can perform manifestly covariant calculation in LF helicity basis to obtain the form factors $G_i(i = 1, 2, 3)$ using dimensional regularization. Here we list only the essential steps for the derivation of the covariant form factors:

(i) We reduce the five propagators into the sum of three propagators using

$$\begin{aligned} \frac{1}{D_\Lambda D_0 D_k D'_0 D'_\Lambda} &= \frac{1}{(\Lambda^2 - m_q^2)^2} \left(\frac{1}{D'_\Lambda} - \frac{1}{D'_0} \right) \\ &\quad \times \frac{1}{D_k} \left(\frac{1}{D_\Lambda} - \frac{1}{D_0} \right), \end{aligned} \quad (23)$$

where

$$\begin{aligned} D_\Lambda &= (k-p)^2 - \Lambda^2 + i\varepsilon, \\ D_0 &= (k-p)^2 - m_q^2 + i\varepsilon, \\ D_k &= k^2 - m_q^2 + i\varepsilon, \end{aligned} \quad (24)$$

and $D'_{0[\Lambda]} = D_{0[\Lambda]}(p \rightarrow p')$.

(ii) We use the Feynman parametrization for the three propagators, e.g.,

$$\begin{aligned} \frac{1}{D_k D_0 D'_0} &= 2 \int_0^1 dx \int_0^{1-x} dy \\ &\quad \times \frac{1}{[D_k(1-x-y) + D_0 x + D'_0 y]^3}, \end{aligned} \quad (25)$$

and compute the integrals over the momentum by shifting the integration variable from k to $k' = k - (xp + yp')$ in

both numerator and denominator which can be written as $D = k'^2 + xp^2 + yp'^2 - (xp + yp')^2 - (x + y)m_q^2 - (1 - x - y)m_d^2$. (iii) We make a Wick rotation of Eq. (21) in D -dimension to regularize the integral.

Following the above procedures (i) -(iii), we finally obtain the covariant form factors. The procedure is very similar to Ref. [22], from which one can find some technical details.

In the present work of LF dynamics (LFD), we shall use only the plus-component $J_{h'h}^+$ of the current matrix elements $J_{h'h}^\mu$ in the calculation of form factors. One can directly calculate the term $S_{h'h}^+$ with $k^- = k_{\text{pole}}^-$, which depends on the integration region of k^+ . However, for the purpose of a clear understanding of the physics implied in LFD, we separate $S_{h'h}^+$ into the on-energy-shell part and the off-energy-shell part by using the following identity

$$\sum_{\lambda_V} \epsilon^\rho(k, \lambda_V) \epsilon^{*\nu}(k, \lambda_V) = -g^{\rho\nu} + \frac{k^\rho k^\nu}{m_d^2} = -g^{\rho\nu} + \frac{[k_{\text{on}}^\rho + \frac{q^{\rho+}}{2}(k^- - k_{\text{on}}^-)][k_{\text{on}}^\nu + \frac{q^{\nu+}}{2}(k^- - k_{\text{on}}^-)]}{m_d^2}, \quad (26)$$

where the metric $g^{+-} = 2$ and the subscript (on) denotes the on-energy-shell ($k^2 = m_q^2$) quark propagator, i.e. $k^- = k_{\text{on}}^- = (m_q^2 + \vec{k}_\perp^2)/k^+$. Then we separate the term $S_{h'h}^+$ into the on-energy-shell propagating part, $(S_{h'h}^+)_{\text{on}}$, and the off-energy-shell part, $(S_{h'h}^+)_{\text{off}}$

$$S_{h'h}^+ = (S_{h'h}^+)_{\text{on}} + (S_{h'h}^+)_{\text{off}}. \quad (27)$$

The off-energy-shell part $(S_{h'h}^+)_{\text{off}}$ can be divided into

$$(S_{h'h}^+)_{\text{off}} = (k^- - k_{\text{on}}^-)(T_{h'h}^+)_{\text{off}} + (k^- - k_{\text{on}}^-)^2(V_{h'h}^+)_{\text{off}}. \quad (28)$$

In the Appendix, we present the explicit expressions of $(S_{h'h}^+)_{\text{on}}$ and $(S_{h'h}^+)_{\text{off}}$ and list the calculated results of each helicity amplitude in DYW frame.

Now, by doing the integration over k^- , one can derive the LF amplitudes from the covariant amplitude in Eq. (21). In terms of LF components one gets

$$\begin{aligned} J_{h'h}^+(0) &= iN_c g_{\Delta} g_P \int \frac{dk^+ dk^- d\vec{k}_\perp^2}{2(2\pi)^4} \\ &\times \frac{1}{k^+(k^+ - p^+)^2(k^+ - p'^+)^2} \\ &\times \frac{1}{(k^- - k_{zm}^-)(k^- - k'_{z\Lambda})} \frac{\Lambda^2 S_{h'h}^+ \Lambda^2}{(k^- - k_{vs}^-)} \\ &\times \frac{1}{(k^- - k_{zm}^-)(k^- - k'_{z\Lambda})}, \end{aligned} \quad (29)$$

where the five poles in k^- are explicitly given by

$$\begin{cases} k_{vs}^- = \frac{m_q^2 + \vec{k}_\perp^2}{k^+} - \frac{i\varepsilon}{k^+}, \\ k_{zm}^- = p^- + \frac{m_q^2 + (\vec{k}_\perp - \vec{p}_\perp)^2}{k^+ - p^+} - \frac{i\varepsilon}{k^+ - p^+}, \\ k'_{zm}^- = p'^- + \frac{m_q^2 + (\vec{p}'_\perp - \vec{k}_\perp)^2}{k^+ - p'^+} - \frac{i\varepsilon}{k^+ - p'^+}, \\ k_{z\Lambda}^- = p^- + \frac{\Lambda^2 + (\vec{k}_\perp - \vec{p}_\perp)^2}{k^+ - p^+} - \frac{i\varepsilon}{k^+ - p^+}, \\ k'_{z\Lambda}^- = p'^- + \frac{\Lambda^2 + (\vec{k}_\perp - \vec{p}'_\perp)^2}{k^+ - p'^+} - \frac{i\varepsilon}{k^+ - p'^+}. \end{cases} \quad (30)$$

Let us now consider the pole structure of Eq. (29) due to only the constituent propagators. As is well known, applying the Cauchy theorem, four different cases should be analyzed: $k^+ < 0$, $0 \leq k^+ \leq p^+$, $p^+ \leq k^+ \leq p'^+$ and $k^+ > p'^+$. The first and fourth cases do not contribute to the integral over k^- , because all the poles in Eq. (29) have imaginary parts with the same sign. It can be easily seen that the only surviving contributions come from the regions $0 \leq k^+ \leq p^+$ and $p^+ \leq k^+ \leq p'^+$. In the former region, the integration over k^- can be done in the lower half-plane, so that only the pole at $k^- = k_{vs}^-$ (the spectator-pole) contributes yielding the valence diagram. In the latter region, the contour in the upper half-plane picks up only the pole at $k^- = k'_{zm}^-$ and $k^- = k'_{z\Lambda}^-$ yielding the so-called nonvalence diagram.

To avoid the complexity of treating double k^- -poles, we re-decompose the product of five energy denominators in Eq. (21) into a sum of four energy denominators:

$$\begin{aligned} \frac{1}{D_\Lambda D_0 D_k D'_0 D'_\Lambda} &= \frac{1}{m_q^2 - \Lambda^2} \frac{1}{D_k} \frac{1}{D'_0} \frac{1}{D_0 D_\Lambda} \\ &+ \frac{1}{\Lambda^2 - m_q^2} \frac{1}{D_k} \frac{1}{D'_\Lambda} \frac{1}{D_0 D_\Lambda}. \end{aligned} \quad (31)$$

Then the first term has only one single pole $k^- = k'_{zm}^-$ and the second term has another single pole $k^- = k'_{z\Lambda}^-$ in the upper half-plane.

Therefore, the covariant diagram shown in Fig. 1(a) is in general equivalent to the sum of the LF valence diagram in Fig. 1(b) and the nonvalence diagram in Fig. 1(c), where $\delta = q^+/p^+ = p'^+/p^+ - 1$. The two LF time-ordered contributions to the residues correspond to the two poles in k^- , the one coming from the interval (I) $0 < k^+ < p^+$ [the “valence contribution”], and the other one

from (II) $p^+ < k^+ < p'^+$ [the “nonvalence contribution” or the “zero mode” when $\delta \rightarrow 0$].

A. Valence Contribution

In the region $0 < k^+ < p^+$ as shown in Fig. 1(b), the pole $k^- = k_{\text{on}}^- = (m_d^2 + \vec{k}_\perp^2 - i\epsilon)/k^+$ (i.e., the spectator quark), is located in the lower half of the complex k^- plane. Thus, the Cauchy integration formula for the k^- -integral in Eq. (29) gives in this region the following result for the plus current;

$$J_{h'h}^+(V) = N_c \int \frac{dx d^2\vec{k}_\perp}{2(2\pi)^3} \frac{S_{h'h}^+(V)}{x(1-x')^2(1-x)^2} \times \frac{g_\Delta \Lambda^2}{(M_\Delta^2 - \mathcal{M}'_0{}^2)(M_\Delta^2 - \mathcal{M}'_\Lambda{}^2)} \times \frac{g_P \Lambda^2}{(M_P^2 - \mathcal{M}'_0{}^2)(M_P^2 - \mathcal{M}'_\Lambda{}^2)}, \quad (32)$$

where $x' = \frac{x}{1+\delta}$, the invariant masses of the initial state are

$$\mathcal{M}_0^2 = \frac{\vec{k}_\perp^2 + m_d^2}{x} + \frac{\vec{k}_\perp^2 + m_q^2}{1-x}, \quad \mathcal{M}_\Lambda^2 = \frac{\vec{k}_\perp^2 + m_d^2}{x} + \frac{\vec{k}_\perp^2 + \Lambda^2}{1-x}, \quad (33)$$

and the invariant masses of the final state are

$$\mathcal{M}'_0{}^2 = \frac{\vec{k}'_\perp{}^2 + m_d^2}{x'} + \frac{\vec{k}'_\perp{}^2 + m_q^2}{1-x'}, \quad \mathcal{M}'_\Lambda{}^2 = \frac{\vec{k}'_\perp{}^2 + m_d^2}{x'} + \frac{\vec{k}'_\perp{}^2 + \Lambda^2}{1-x'}. \quad (34)$$

Here, $\vec{k}'_\perp = \vec{k}_\perp - x'\vec{q}_\perp$ is the Drell-Yan assignment [12].

Due to $\gamma^+\gamma^+ = 0$ and the pole k_{on}^- , both quark and diquark are on-energy-shell. As one can easily see from Eq.(28), $S_{h'h}^{+val} = (S_{h'h}^+)_{\text{on}}$. Therefore, the physical interpretation of the LF plus current matrix element given by Eq. (32) is manifest in terms of LF wave functions, i.e., a convolution of the initial and final state LF wavefunctions. Such identification is not possible for the covariant calculation. From Eq. (32), the initial and the final state LF wave functions are the smeared LF wavefunctions given by

$$\varphi_\Delta(x', \vec{k}'_\perp) = \frac{g_\Delta \Lambda^2}{(M_\Delta^2 - \mathcal{M}'_0{}^2)(M_\Delta^2 - \mathcal{M}'_\Lambda{}^2)}, \quad \varphi_P(x, \vec{k}_\perp) = \frac{g_P \Lambda^2}{(M_P^2 - \mathcal{M}'_0{}^2)(M_P^2 - \mathcal{M}'_\Lambda{}^2)}. \quad (35)$$

B. Zero-mode contribution

In the region $p^+ < k^+ < p'^+ (= p^+ + q^+)$ as shown in Fig. 1(c), the poles are at $k^- = p'^- + \frac{(\vec{k}_\perp - \vec{p}'_\perp)^2 + m_q^2 - i\epsilon}{k^+ - p'^+}$ (from the struck quark propagator) and $k^- = p'^- + \frac{(\vec{k}_\perp - \vec{p}'_\perp)^2 + \Lambda^2 - i\epsilon}{k^+ - p'^+}$ (from the smeared quark-photon vertex $S_\Lambda(k - p')$). Both of them are located in the upper half plane of the complex k^- space.

When we do the Cauchy integration over k^- to obtain the LF time-ordered diagrams, we decompose the product of five energy denominators into a sum of two terms in Eq. (31), i.e.,

$$J_{h'h}^+(Z) = J_{h'h}^+(Z, k_{zm}^-) + J_{h'h}^+(Z, k_{z\Lambda}^-). \quad (36)$$

After above decomposition, the first term in Eq. (36) that has only one pole $k^- = k_{zm}^-$ is given by

$$J_{h'h}^+(Z, k_{zm}^-) = \frac{N_c \Lambda^4}{(\Lambda^2 - m_q^2)} \int_1^{1+\delta} \frac{dx d^2\vec{k}_\perp}{2(2\pi)^3} \times \frac{g_\Delta}{(M_\Delta^2 - \mathcal{M}'_0{}^2)} \frac{S_{h'h}^+(Z, k^- = k_{zm}^-)}{x'(x-1)^2(x'-1)} \times \frac{g_P}{(q^2 - \mathcal{M}'_{qm}{}^2)(q^2 - \mathcal{M}'_{q\Lambda}{}^2)}, \quad (37)$$

where $q^2 = \vec{q}_\perp^2 + M_\Delta^2 - M_P^2$ and the invariant masses are defined as

$$\mathcal{M}'_{qm}{}^2 = \frac{(\vec{k}_\perp - \vec{q}_\perp)^2 + m_q^2}{1-x'} + \frac{\vec{k}_\perp^2 + m_q^2}{(x-1)/(1+\delta)}, \quad \mathcal{M}'_{q\Lambda}{}^2 = \frac{(\vec{k}_\perp - \vec{q}_\perp)^2 + m_q^2}{1-x'} + \frac{\vec{k}_\perp^2 + \Lambda^2}{(x-1)/(1+\delta)}. \quad (38)$$

Here, the initial state wavefunction is the photon smeared LF wave function

$$\varphi_q(x, \vec{k}_\perp; q^2) = \frac{\Lambda^2}{(q^2 - \mathcal{M}'_{qm}{}^2)(q^2 - \mathcal{M}'_{q\Lambda}{}^2)}. \quad (39)$$

The second term in Eq. (36) that also has a single pole $k^- = k_{z\Lambda}^-$ is given by

$$J_{h'h}^+(Z, k_{z\Lambda}^-) = \frac{N_c \Lambda^4}{(\Lambda^2 - m_q^2)} \int_1^{1+\delta} \frac{dx d^2\vec{k}_\perp}{2(2\pi)^3} \times \frac{g_\Delta}{(M_\Delta^2 - \mathcal{M}'_0{}^2)} \frac{S_{h'h}^+(Z, k^- = k_{z\Lambda}^-)}{x'(x-1)^2(1-x')} \times \frac{g_P}{(q^2 - \mathcal{M}'_{\Lambda m}{}^2)(q^2 - \mathcal{M}'_{\Lambda\Lambda}{}^2)}, \quad (40)$$

where the invariant masses are

$$\mathcal{M}'_{\Lambda m}{}^2 = \frac{(\vec{k}_\perp - \vec{q}_\perp)^2 + \Lambda^2}{1-x'} + \frac{\vec{k}_\perp^2 + m_q^2}{(x-1)/(1+\delta)}, \quad \mathcal{M}'_{\Lambda\Lambda}{}^2 = \frac{(\vec{k}_\perp - \vec{q}_\perp)^2 + \Lambda^2}{1-x'} + \frac{\vec{k}_\perp^2 + \Lambda^2}{(x-1)/(1+\delta)}. \quad (41)$$

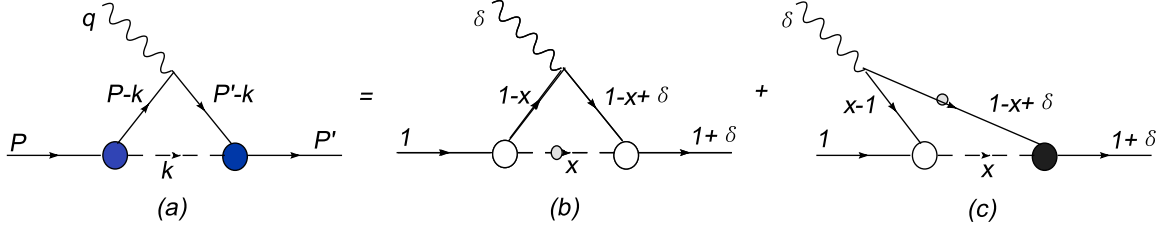


FIG. 1: The covariant triangle diagram (a) is represented as the sum of a LF valence diagram (b) defined in the region $0 < k^+ < p^+$ and the nonvalence diagram (c) defined in $p^+ < k^+ < p'^+$. $\delta = q^+/p^+ = p'^+/p^+ - 1$. The white and black blobs at the baryon-quark vertices in (b) and (c) represent the LF wavefunction and non-wavefunction vertices, respectively. The small circles in (b) and (c) represent the (on-shell) mass pole of the quark propagator determined from the k^- -integration.

Adding Eqs. (37) and (40), we get the full result in nonvalence region. In general, the terms in $S_{h'h}^+$ include the on-energy-shell and off-energy-shell contributions in the nonvalence region; *e.g.*, $S_{h'h}^+(k^-) = (S_{h'h}^+)_{\text{on}} + (S_{h'h}^+)_{\text{off}}(k^-)$. In addition, although the nonvalence part of transition form factors can not be directly written as the convolution of the conventional wavefunctions like that of the valence part, we can still utilize the LF Bethe-Salpeter approach and relate the nonvalence part to the valence part [23]. This allows the convolution formalism also for the nonvalence part. Once both valence and nonvalence parts can be expressed as the convolution formulism, we can replace these smeared LF wavefunctions by the conventional LFQM Gaussian wavefunctions and recover the usual LFQM description.

In the $q^+ \rightarrow 0$ limit, the integration range of the nonvalence region, $p^+ < k^+ < p'^+ (= p^+ + q^+)$, shrinks to zero so that the nonvalence contribution reduces to the zero mode contribution [2, 3, 4, 5]

$$(J_{h'h}^+)_{\text{z.m.}} = \lim_{\delta \rightarrow 0} (J_{h'h}^+)_{\text{nv}} = \lim_{\delta \rightarrow 0} \int_1^{1+\delta} dx (\cdots). \quad (42)$$

The non-vanishing zero-mode contribution occurs only if the integrand (\cdots) in Eq. (42) behaves as $\sim k^-$ (i.e. $(1-x)^{-1}$). Note that there is no zero-mode contribution in the case that the integrand is k^- -independent or behaves like $k^-(k^+ - p^+)^n$ ($n \geq 1$). For the plus current, the zero-mode contribution comes from the helicity matrix elements of the propagators $S_{h'h}^+(k^-)$, specifically only from the instantaneous part, and neither from the on-energy-shell propagating part nor the energy denominator. In the Appendix, we analyze the zero mode contribution

for each helicity element by using the power counting method [5].

Performing the calculation in Eq. (37), the result for $J_{h'h}^+(Z, k_{zm}')$ is given by

$$J_{h'h}^+(Z, k_{zm}') = \frac{N_c g_\Delta g_P \Lambda^4}{(\Lambda^2 - m_q^2)} \int_1^{1+\delta} \frac{dx d^2 \vec{k}_\perp}{2(2\pi)^3} \frac{1}{\mathcal{B}_m \mathcal{B}_\Lambda} \times \left[\frac{(x-1-\delta)}{x} T_{h'h}^+ + \frac{1}{x^2} \mathcal{A} V_{h'h}^+ \right], \quad (43)$$

where

$$\mathcal{A} = \frac{M_\Delta^2 + \vec{q}_\perp^2}{1+\delta} (x-1-\delta)x + [(\vec{k}_\perp - \vec{q}_\perp)^2 + m_q^2]x - [\vec{k}_\perp^2 + m_d^2](x-1-\delta), \quad (44)$$

and

$$\begin{aligned} \mathcal{B}_m &= \left(\frac{M_\Delta^2 + \vec{q}_\perp^2}{1+\delta} - M_P^2 \right) (x-1)(x-1-\delta) \\ &\quad + [(\vec{k}_\perp - \vec{q}_\perp)^2 + m_q^2](x-1) \\ &\quad - [\vec{k}_\perp^2 + m_q^2](x-1-\delta), \\ \mathcal{B}_\Lambda &= \left(\frac{M_\Delta^2 + \vec{q}_\perp^2}{1+\delta} - M_P^2 \right) (x-1)(x-1-\delta) \\ &\quad + [(\vec{k}_\perp - \vec{q}_\perp)^2 + m_q^2](x-1) \\ &\quad - [\vec{k}_\perp^2 + \Lambda^2](x-1-\delta). \end{aligned} \quad (45)$$

Similarly, we can get the result for $J_{h'h}^+(Z, k_{z\Lambda}')$. If we write $x = 1 + \delta y$ and $dx = \delta dy$ in Eq. (43) and $J_{h'h}^+(Z, k_{z\Lambda}')$, then the integral over y runs from 0 to 1 as x runs from 1 to $1+\delta$. Taking $q^+ \rightarrow 0$ limit ($\delta \rightarrow 0$), the zero mode contribution is given by

$$\begin{aligned}
(J_{h'h}^+(Z))_{\text{z.m.}} &= \frac{N_c g_\Delta g_p \Lambda^4}{(\Lambda^2 - m_q^2)} \int_0^1 \frac{dy d^2 \vec{k}_\perp}{2(2\pi)^3} \\
&\times \left\{ \frac{-(U_{h'h}^+)_{\text{off}}[(\vec{k}_\perp - \vec{q}_\perp)^2 + m_q^2]y - (T_{h'h}^+)_{\text{off}}(1-y)}{\left[((\vec{k}_\perp - \vec{q}_\perp)^2 + m_q^2)y + (\vec{k}_\perp^2 + m_q^2)(1-y) \right] \left[((\vec{k}_\perp - \vec{q}_\perp)^2 + m_q^2)y + (\vec{k}_\perp^2 + \Lambda^2)(1-y) \right]} \right. \\
&\left. + \frac{(U_{h'h}^+)_{\text{off}}[(\vec{k}_\perp - \vec{q}_\perp)^2 + \Lambda^2]y + (T_{h'h}^+)_{\text{off}}(1-y)}{\left[((\vec{k}_\perp - \vec{q}_\perp)^2 + \Lambda^2)y + (\vec{k}_\perp^2 + \Lambda^2)(1-y) \right] \left[((\vec{k}_\perp - \vec{q}_\perp)^2 + \Lambda^2)y + (\vec{k}_\perp^2 + m_q^2)(1-y) \right]} \right\}. \quad (46)
\end{aligned}$$

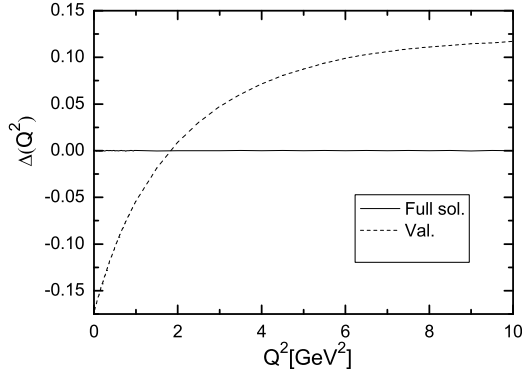


FIG. 2: The solid curve represents the angular conditions $\Delta(Q^2)$ including the zero mode while the dash curve is the angular condition excluding the zero mode. $\Delta(Q^2)$ equals to zero exactly after considering zero mode.

where $(U_{h'h}^+)_{\text{off}}$ corresponds to $\frac{(V_{h'h}^+)_{\text{off}}}{1-x}$. As shown in the Appendix, $(J_{\frac{1}{2},\frac{1}{2}}^+(Z))_{\text{z.m.}}$ and $(J_{\frac{1}{2},-\frac{1}{2}}^+(Z))_{\text{z.m.}}$ do not vanish while the others $((J_{\frac{3}{2},\frac{1}{2}}^+(Z))_{\text{z.m.}}$ and $(J_{\frac{3}{2},-\frac{1}{2}}^+(Z))_{\text{z.m.}}$) do.

V. NUMERICAL RESULTS

In this section, we present the numerical results for the transition form factors and verify that all of the form factors obtained in the LF helicity basis are in complete agreement with the manifestly covariant results. However, we do not aim at finding the best-fit parameters to describe the experimental data of the N - Δ transition properties. Rather, we simply take typical parameters used before. Nevertheless, our model calculations have a generic structure and the essential findings from our calculations may apply to the more realistic models. The

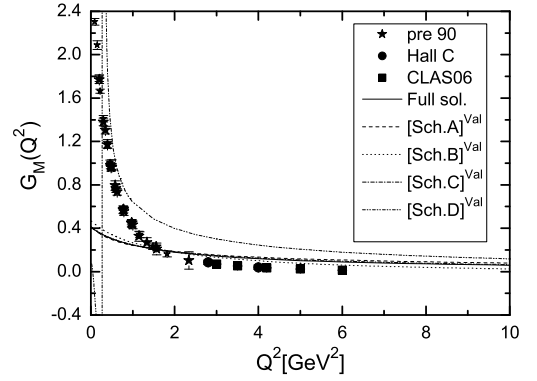


FIG. 3: The magnetic form factor $G_M(Q^2)$. The filled circles correspond to JLab Hall C data [24]. The filled squares are from CLAS experiment [25]. The filled stars are from pre-90 data at DESY, Bonn and SLAC [26, 27, 28, 29, 30]. The solid curve represents the full solutions of the form factor $G_M(Q^2)$ in four schemes which have the same curve. The dash curve represents the valence part of the form factors in scheme A, the dot curve in scheme B, the dash-dot curve in scheme C and the dash-dot-dot curve in scheme D.

quantitative results would certainly depend on the details of the model.

In our numerical calculations, we thus use the quark and diquark masses as $m_q = 0.55$ GeV and $m_d = 0.70$ GeV, respectively, noting that a rather large value of quark mass is allowed to handle the Δ particle and take $\Lambda = 1.8$ GeV as in the calculation of spin-one meson form factors [2]. Also, we simply take the values of parameters g_P and g_Δ as typical strong coupling constant 1. In this work, we do not consider the flavor structure of nucleon and $\Delta(1232)$ to show that the zero-mode issue is completely independent from whether the model is “spectator diquark” or “struck diquark”. In addition, we make

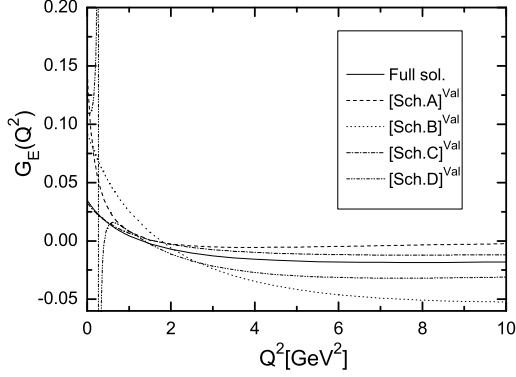


FIG. 4: The electric form factor $G_E(Q^2)$ in three different schemes with and without zero mode. The five curves correspond to the same cases as in Fig. 3.

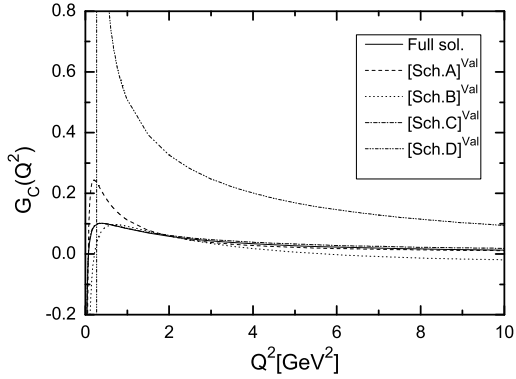


FIG. 5: The Coulomb form factor $G_C(Q^2)$ in three different schemes with and without zero mode. The five curves correspond to the same cases as in Fig. 3.

the taxonomical decompositions of the full results into the valence and nonvalence contributions to facilitate a quantitative comparison between the full results and the valence parts of the form factors (G_M, G_C, G_E) in four different schemes. To show the importance of zero mode, we also compare the angular condition $\Delta(Q^2)$ including the zero mode with the one without the zero mode.

In Fig. 2, we show the angular condition $\Delta(Q^2)$ given by Eq. (9), neglecting the zero-mode contribution. From the dashed curve in the Fig. 2, we find that $\Delta(Q^2) \neq 0$ and thus the angular condition is violated. If we include the zero-mode contribution, $\Delta(Q^2)$ is exactly zero in all four schemes as shown by the solid curve. Thus, the

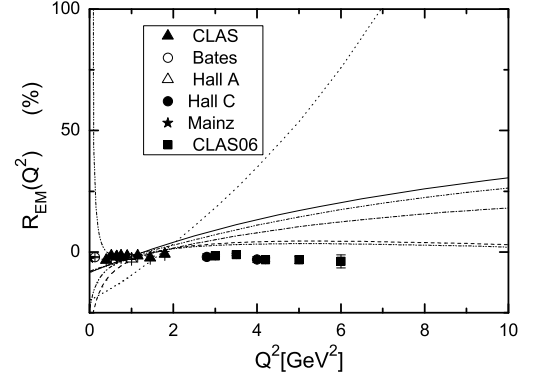


FIG. 6: The ratios R_{EM} . Both the filled squares [25] and filled triangles [31] are the CLAS results. The open triangles are from an JLab Hall A experiment [32]. The filled circles: JLab Hall-C [24]. Open circles: Bates [33]. Filled stars: Mainz [34]. The five curves correspond to the same cases as in Fig. 3.

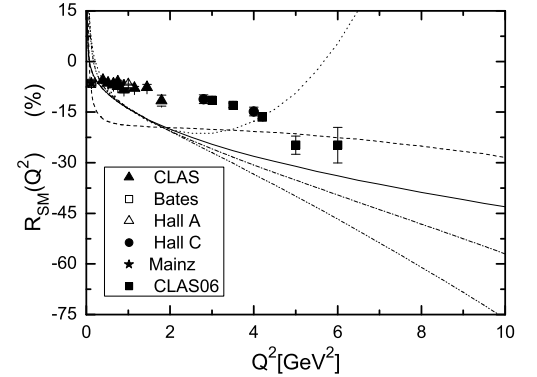


FIG. 7: The ratios R_{SM} . Both the filled squares [25] and filled triangles [31] are the CLAS results. The open triangles are from an JLab Hall A experiment [32]. The filled circles: JLab Hall-C [24]. Open circles: Bates [33]. Filled stars: Mainz [35]. The five curves correspond to the same cases as in Fig. 3.

zero mode contribution is crucial to get the correct full solution.

The N - Δ transition is given predominantly in the magnetic dipole (M1) type. In the instant-form quark model picture, the N - Δ transition is described by a spin flip of a quark in the s-wave state, which leads to the magnetic dipole (M1) type transition. In LFQM, because of the Melosh transformation, the relativistic effect is also included. We show the magnetic form factor G_M in Fig. 3.

The full solutions (solid curve) are obtained from four different prescriptions and they all turn out to give exactly the same result as the covariant one. To estimate the zero mode contribution, we also plot the valence contribution for each prescription, i.e., the dashed curve for scheme A, dotted curve for scheme B, dash-dotted curve for scheme C and dash-dot-dotted curve for scheme D, respectively. In particular, the valence contribution of scheme D yields a singular behavior near $Q^2 = (M_\Delta - M_N)M_N$ due to the denominator structure K_D in Eq.(13). However, the full solution of scheme D including the zero mode contribution renders the regular result depicted by the solid curve coinciding with the full solutions of other schemes (A,B and C) as expected. Because we simply take the typical parameters, and consider neither the flavor structure nor the struck-diquark contribution, it is not really surprising to see a rather large disagreement of our present model calculation from the experimental data as shown in Fig. 3. The LF wavefunction we took in the present model calculation is also not realistic since it is just coming from the smearing procedure of the triangle loop. The purpose of our present analysis is not to reproduce the data, but to show the necessity of taking into account the zero mode contribution to get the correct result without any scheme dependence. Our analysis here also indicates that the zero-mode issue is completely independent from whether the model is “spectator diquark” or “struck diquark”. In another words, the Lorentz covariance must work independently for each contribution whether the struck constituent is quark or diquark.

The electric and Coulomb form factors $G_E(Q^2)$ and $G_C(Q^2)$ are shown in Figs. 4 and 5, respectively. As anticipated, the full solution in each prescription is exactly the same as the covariant results. However, unlike the case of G_M the valence contributions of $G_E(Q^2)$ and $G_C(Q^2)$ to the full solution are quite different even in the schemes of A, B and C although the scheme D has already been expected to be different due to the denominator structure K_D in Eq.(13). This suggests that G_C and G_E are more sensitive to the zero mode than G_M . A very interesting point is that the values of both G_E and G_C are not zero at $Q^2 = 0$ although the s-wave instant-form SU(6) quark model predicts that the values of both G_E and G_C are exactly zero in non-relativistic limit [36]. The starting point of our model is also the s-wave instant-form SU(6) quark wavefunction. On the other hand, in LFQM the relativistic effect and orbital angular momentum [15] are responsible for the electric

(E2) and Coulomb (C2) quadrupole transitions. It has been shown that there are indeed different mechanisms to produce non-zero values of G_E and G_C . Even in the non-relativistic quark model (excluding the s-wave SU(6) breaking model), the d-wave admixture in the nucleon and Δ wave functions allows for the electric (E2) and Coulomb (C2) quadrupole transitions.

These physical form factors $G_M(Q^2)$, $G_E(Q^2)$ and $G_C(Q^2)$ can be related to other physical quantities $R_{EM}(Q^2)$ and $R_{SM}(Q^2)$. The relations are as follows [10, 11]

$$\begin{aligned} R_{EM}(Q^2) &= \frac{E2(Q^2)}{M1(Q^2)} = \frac{E_{1+}}{M_{1+}} = -\frac{G_E(Q^2)}{G_M(Q^2)}, \\ R_{SM}(Q^2) &= \frac{C2(Q^2)}{M1(Q^2)} = \frac{S_{1+}}{M_{1+}} = -\frac{Q_+Q_-}{4M_\Delta^2} \frac{G_C(Q^2)}{G_M(Q^2)}, \end{aligned} \quad (47)$$

where $Q_\pm = \sqrt{(M_\Delta \pm M_P)^2 + Q^2}$. In Fig. 6, we show the physical quantity R_{EM} obtained by using the light-front helicity basis both from full solutions and from only valence part of physical form factors in four (A,B,C,D) schemes. The corresponding results for the physical quantity R_{SM} are shown in Fig. 7. In Fig. 6, the calculated value of R_{EM} at $Q^2 = 0$ is -8.33% . In non-relativistic quark models with unbroken SU(6)-spin-flavour symmetry, however, R_{EM} is predicted to be exactly zero [36], whereas a broken SU(6) symmetry yields the values ranging from 0 to -0.2% [37]. It is also known well that the perturbative QCD (pQCD) power counting rule predicts that R_{EM} approaches to 1 [38] and R_{SM} approaches to constant up to logarithmic corrections [39] in the limit $Q^2 \rightarrow \infty$. The 12 GeV upgrade of JLab facility is anticipated to shed some light on the applicability of PQCD at large Q^2 . For the better agreement between our model calculation and the experimental data of G_M , R_{EM} and R_{SM} , we need to consider the flavor structure and use the more realistic Gaussian LF wavefunctions.

VI. CONCLUSION

In this work, we investigated the electromagnetic transition between nucleon and $\Delta(1232)$ using both the manifestly covariant technique in the helicity basis and the light-front (LF) calculation for the matrix elements of the plus component current. In the LF calculation, the full solution is decomposed into the valence and zero-mode contribution. Using the power counting method and exact calculation for each helicity amplitude, we

found that only two helicity amplitudes receive the zero mode contribution. From the numerical computation, we have explicitly shown that the full results of the LF calculation including the zero mode contribution are in complete agreement with the covariant one without prescription dependence. Our analysis also showed that the zero-mode issue is completely independent from whether the model is “spectator diquark” or “struck diquark” and the Lorentz covariance must work independently for each contribution whether the struck constituent is quark or diquark. Because our model calculations have a generic structure, i.e., a convolution of initial and final state LF wavefunctions, our calculations can apply to more realistic cases. The treatment for the zero mode contribution

can also be extended to other nucleon-resonance transition processes.

Acknowledgments

This work is partially supported by National Natural Science Foundation of China (Nos. 10421503, 10575003, 10528510), by the Key Grant Project of Chinese Ministry of Education (No. 305001), and by the Research Fund for the Doctoral Program of Higher Education (China). CRJ’s work was supported by a grant from the U.S. Department of Energy under Contract No. DE-FG02-03ER41260.

APPENDIX: SUMMARY OF SPINOR TERMS IN HELICITY AMPLITUDE

The on-energy-shell part $(S_{h'h}^+)_{\text{on}}$ is given by $(S_{h'h}^+)_{\text{on}} = (S_{1h'h}^+)_{\text{on}} + (S_{2h'h}^+)_{\text{on}}$, where

$$\begin{aligned} (S_{1h'h}^+)_{\text{on}} &= -\bar{u}_\rho(p', h')(\not{p}' - \not{k} + m)\gamma^+(\not{p}' - \not{k} + m)\gamma^\rho\gamma_5 u(p, h), \\ (S_{2h'h}^+)_{\text{on}} &= \frac{1}{m_d^2}\bar{u}_\rho(p', h')k_{\text{on}}^\rho(\not{p}' - \not{k} + m)\gamma^+(\not{p}' - \not{k} + m)\not{k}_{\text{on}}\gamma_5 u(p, h), \end{aligned} \quad (\text{A.1})$$

and the off-energy-shell part $(S_{h'h}^+)_{\text{off}}$ is given by

$$\begin{aligned} (S_{h'h}^+)_{\text{off}} &= (k^- - k_{\text{on}}^-)(T_{h'h}^+)_{\text{off}} + (k^- - k_{\text{on}}^-)^2(V_{h'h}^+)_{\text{off}} \\ &= (k^- - k_{\text{on}}^-)(T_{1h'h}^+)_{\text{off}} + (k^- - k_{\text{on}}^-)(T_{2h'h}^+)_{\text{off}} + (k^- - k_{\text{on}}^-)^2(V_{h'h}^+)_{\text{off}}, \end{aligned} \quad (\text{A.2})$$

where

$$\begin{aligned} (T_{1h'h}^+)_{\text{off}} &= \frac{1}{m_d^2}\bar{u}_-(p', h')(\not{p}' - \not{k} + m)\gamma^+(\not{p}' - \not{k} + m)\not{k}_{\text{on}}\gamma_5 u(p, h), \\ (T_{2h'h}^+)_{\text{off}} &= \frac{1}{m_d^2}\bar{u}_\rho(p', h')k_{\text{on}}^\rho(\not{p}' - \not{k} + m)\gamma^+(\not{p}' - \not{k} + m)\gamma_-\gamma_5 u(p, h), \\ (V_{h'h}^+)_{\text{off}} &= \frac{1}{m_d^2}\bar{u}_-(p', h')(\not{p}' - \not{k} + m)\gamma^+(\not{p}' - \not{k} + m)\gamma_-\gamma_5 u(p, h). \end{aligned} \quad (\text{A.3})$$

In DYW frame with the LF gauge, the explicit forms of the spinor terms in Eqs. (A.1) and (A.2) for each helicity component are given as follows.

(I) helicity $(\frac{3}{2}, \frac{1}{2})$ -component:

$$\begin{aligned} (S_{\frac{3}{2}, \frac{1}{2}}^+)_{\text{on}} &= [k^L - xq^L] \{x^2 M_P m_q [m_q + (1-x)M_\Delta] + xm_d^2 [m_q - (1-x)M_\Delta + 2(1-x)M_P] \\ &\quad - k^L k^R [(1-x)M_\Delta + (1-x)m_q - xM_P] - x^2 q^L k^R (m_q + M_P)\} \\ &\quad - [k^L + xq^L] m_d^2 [m_q + (1-x)M_\Delta], \end{aligned} \quad (\text{A.4})$$

where $k^{R(L)} = k^1 \pm ik^2$ and

$$\begin{aligned} (T_{\frac{3}{2}, \frac{1}{2}}^+)_{\text{off}} &= 0, \\ (T_{\frac{3}{2}, \frac{1}{2}}^+)_{\text{off}} &= \frac{2\sqrt{2}(p^+)^2}{m_d^2}(1-x)(k^L - xq^L)[m_q + (1-x)M_\Delta], \\ (V_{\frac{3}{2}, \frac{1}{2}}^+)_{\text{off}} &= 0. \end{aligned} \quad (\text{A.5})$$

In the nonvalence region where $k^- \sim 1/(1-x)$, we find that all the off-shell terms are regular as $q^+ \rightarrow 0$ (or equivalently $x \rightarrow 1$). Therefore, there is no zero-mode in the helicity $(\frac{3}{2}, \frac{1}{2})$ component.

(II) helicity $(\frac{3}{2}, -\frac{1}{2})$ -component:

$$(S_{\frac{3}{2}, -\frac{1}{2}}^+)_{\text{on}} = [k^L - xq^L] \{k^L k^R (k^L - xq^L) + (1-x)m_d^2(k^L + xq^L) + xm_q(x^2 M_P q^L + m_q k^L) + x(1-x)k^L(M_P M_\Delta + m_q M_P + m_q M_\Delta)\}, \quad (\text{A.6})$$

and

$$\begin{aligned} (T_{1\frac{3}{2}, -\frac{1}{2}}^+)_{\text{off}} &= 0, \\ (T_{2\frac{3}{2}, -\frac{1}{2}}^+)_{\text{off}} &= \frac{-2\sqrt{2}(p^+)^2}{m_d^2}(1-x)(k^L - xq^L)^2, \\ (V_{\frac{3}{2}, -\frac{1}{2}}^+)_{\text{off}} &= 0. \end{aligned} \quad (\text{A.7})$$

Again, the off-shell terms are regular as $x \rightarrow 1$. Therefore, there is no zero-mode in the helicity $(\frac{3}{2}, -\frac{1}{2})$ component.

(III) helicity $(\frac{1}{2}, \frac{1}{2})$ -component:

$$\begin{aligned} (S_{1\frac{1}{2}, \frac{1}{2}}^+)_{\text{on}} &= 4\sqrt{\frac{2}{3}} \frac{p^+}{M_\Delta} \{[m_q M_P + (1-x)M_\Delta^2 - q^L q^R][m_q + (1-x)M_\Delta] \\ &\quad + [M_P k^R + (1-x)(M_P - M_\Delta)q^R][k^L - xq^L]\} \\ &\quad - 4\sqrt{\frac{2}{3}} p^+ \{(1-x)q^L + k^L[k^R - xq^R] + [m_q + (1-x)M_P][m_q + (1-x)M_\Delta]\}, \end{aligned} \quad (\text{A.8})$$

$$\begin{aligned} (S_{2\frac{1}{2}, \frac{1}{2}}^+)_{\text{on}} &= -4\sqrt{\frac{1}{6}} \frac{p^+}{m_d^2 x^2} [m_d^2 - x^2 M_\Delta^2 + (k^R - xq^R)(k^L - xq^L)] \\ &\quad \times \{[m_q + (1-x)M_\Delta][x^2 M_P m_q - (1-x)m_d^2 - k^L k^R] + (k^L - xq^L)k^R x(m_q + M_P)\} \\ &\quad - 4\sqrt{\frac{1}{6}} \frac{p^+}{m_d^2 x} (k^L - xq^L) \{(-x^2 m_q M_P + (1-x)m_d^2 + k^L k^R)(k^R - q^R x) \\ &\quad + k^R x(M_P + m_q)(m_q + (1-x)M_\Delta)\}, \end{aligned} \quad (\text{A.9})$$

and

$$\begin{aligned} (T_{1\frac{1}{2}, \frac{1}{2}}^+)_{\text{off}} &= -2\sqrt{\frac{2}{3}} \frac{(p^+)^2}{xm_d^2 M_\Delta} \{[m_q + (1-x)M_\Delta][x^2 M_P m_q - (1-x)m_d^2] \\ &\quad + k^L k^R [xM_P - (1-x)m_q - (1-x)M_\Delta] - k^R q^L x^2(m_q + M_P)\}, \\ (T_{2\frac{1}{2}, \frac{1}{2}}^+)_{\text{off}} &= 2\sqrt{\frac{2}{3}} \frac{(p^+)^2}{xm_d^2 M_\Delta} (1-x)[m_q + (1-x)M_\Delta][m_d^2 - x^2 M_\Delta^2 + (k^L - xq^L)(k^R - xq^R)] \\ &\quad - 2\sqrt{\frac{2}{3}} \frac{(p^+)^2}{m_d^2} (1-x)(k^L - xq^L)(k^R - xq^R), \\ (V_{\frac{1}{2}, \frac{1}{2}}^+)_{\text{off}} &= 2\sqrt{\frac{2}{3}} \frac{(p^+)^3}{m_d^2 M_\Delta} (1-x)[m_q + (1-x)M_\Delta]. \end{aligned} \quad (\text{A.10})$$

From the power counting method of the longitudinal momentum fraction, one can see that both $(T_{1\frac{1}{2}, \frac{1}{2}}^+)_{\text{off}}$ and $(V_{\frac{1}{2}, \frac{1}{2}}^+)_{\text{off}}$ yield a singular behavior in the integrand of Eq.(43) giving effectively the zero-mode contribution to the helicity $(\frac{1}{2}, \frac{1}{2})$ component.

(IV) helicity $(\frac{1}{2}, -\frac{1}{2})$ -component:

$$\begin{aligned} (S_{1\frac{1}{2}, -\frac{1}{2}}^+)_{\text{on}} &= 4\sqrt{\frac{2}{3}} \frac{p^+}{M_\Delta} \{q^L(M_P + m_q)[m_q + (1-x)M_\Delta] \\ &\quad + [(k^R + (1-x)q^R)q^L + (1-x)M_P M_\Delta - (1-x)M_\Delta^2][k^L - xq^L]\} \\ &\quad - 4\sqrt{\frac{2}{3}} p^+ (1-x)[m_q + (1-x)M_\Delta]q^L, \end{aligned} \quad (\text{A.11})$$

$$\begin{aligned}
(S_{2\frac{1}{2},-\frac{1}{2}}^+)_\text{on} &= -4p^+ \sqrt{\frac{1}{6}} \frac{1}{x^2 m_d^2} [m_d^2 - x^2 M_\Delta^2 + (k^R - xq^R)(k^L - xq^L)] \\
&\times \{(-x^2 m_q M_P + (1-x)m_d^2 + k^L k^R)(k^L - q^L x) + k^L x(M_P + m_q)(m_q + (1-x)M_\Delta)\} \\
&+ 4p^+ \sqrt{\frac{1}{6}} \frac{1}{x m_d^2} (k^L - xq^L) \{[m_q + (1-x)M_\Delta][x^2 M_P m_q - (1-x)m_d^2 - k^L k^R] \\
&+ (k^R - xq^R)xk^L(m_q + M_P)\}, \tag{A.12}
\end{aligned}$$

and

$$\begin{aligned}
(T_{1\frac{1}{2},-\frac{1}{2}}^+)_\text{off} &= -2\sqrt{\frac{2}{3}} \frac{(p^+)^2}{x m_d^2 M_\Delta} \{q^L x[x^2 m_q M_P - (1-x)m_d^2 - k^L k^R] \\
&+ k^L[(1-x)(m_d^2 + x m_q M_P) + (1+\delta)(k^L k^R + m_q^2) + x(1-x)M_\Delta(m_q + M_P)]\}, \\
(T_{2\frac{1}{2},-\frac{1}{2}}^+)_\text{off} &= 2\sqrt{\frac{2}{3}} \frac{(p^+)^2}{x m_d^2 M_\Delta} (1-x)(k^L - xq^L)[m_d^2 - x^2 M_\Delta^2 + k^L - xq^L](k^R - xq^R) \\
&- 2\sqrt{\frac{2}{3}} \frac{(p^+)^2}{m_d^2} (1-x)(k^L - xq^L)[m_q + (1-x)M_\Delta], \\
(V_{\frac{1}{2},-\frac{1}{2}}^+)_\text{off} &= -2\sqrt{\frac{2}{3}} \frac{(p^+)^3}{m_d^2 M_\Delta} (1-x)(k^L - xq^L). \tag{A.13}
\end{aligned}$$

Again from the power counting method, $(T_{1\frac{1}{2},-\frac{1}{2}}^+)_\text{off}$ and $(V_{\frac{1}{2},-\frac{1}{2}}^+)_\text{off}$ cause a singular behavior in the integrand of Eq.(43) giving again effectively the zero-mode contribution to the helicity $(\frac{1}{2}, -\frac{1}{2})$ component.

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